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EVALUATION OF STATISTICAL ENERGY ANALYSIS PARAMETERS THROUGH FINITE ELEMENT METHOD APPLIED TO STRUCTURES EMPLOYED IN NAVAL APPLICATIONS

José Manoel Freire Guimarães
Vibrations and Acoustics Group - GVA, Federal University of Pará, Belém, PA 66075-110, Brazil
jmfguimaraes@outlook.com

Gabriel Soares Quixaba
Vibrations and Acoustics Group - GVA, Federal University of Pará, Belém, PA 66075-110, Brazil
gqsmsoares@gmail.com

Newton Sure Soeiro
Vibrations and Acoustics Group - GVA, Federal University of Pará, Belém, PA 66075-110, Brazil
nsoeiro@ufpa.br

Leopoldo Pacheco Bastos
Research Group in Acoustics, Thermal Fluid Dynamic and Mechanical Systems - OMNIWAVE, Federal University of Pará, Tucuruí, PA 68464-000, Brazil
leopbastos@ufpa.br

Abstract. *When dealing with vibrational problems at high frequencies or structures with high modal density such as those composed of plates, deterministic approaches such as Finite Element Method (MEF) or Boundary Elements Method (BEM) become impracticable due to computational requirements related to model discretization and error propagation issues. Such problems are of major importance in various fields of engineering within applications in the aerospace, aeronautical, and naval industries. In this context, it is important to use alternative approaches in order to overcome the difficulties presented by traditional methods. Statistical Energy Analysis (SEA) uses statistical methods, which allow the calculation of parameters, such as the coupling loss factor. This parameter is very useful for design of structures in which it is desirable to reduce vibration transmission. Therefore, the present work aims on the numerical analysis of the coupling loss factor for several plate arrangements. To achieve this, MEF and SEA approaches are used. Such analyzes were carried out with the purpose of identifying the arrangements that reduce the vibrational energy transmission between interest components. Those configurations are present in naval applications, such as in engine supports and brackets used in boats.*

Keywords: *Finite Elements Method, Statistical Energy Analysis, Coupling Loss Factor*

1. INTRODUCTION

The transmission of vibratory energy in structures is a recurrent theme in aerospace, automotive and marine industries. This is because energy coming from a particular source can propagate through structures and can result in excessive noise and vibration levels. For this reason, one of the main research focuses of those industries is to determine the transmission path of the vibratory energy and, consequently, to develop means of minimizing it, either by the use of viscoelastic materials, when applied to the structure, or changes in the original structural arrangement (PANKAJ and MURIGENDRAPPA, 2016).

The Finite Element Method (FEM) is a frequently used tool to obtain the structural stresses and vibrational behavior at low frequencies. However, this method presents limitations when predicting phenomena which are strongly influenced by the contribution of many natural frequencies and vibrational modes, such as the problems at high frequency range (WILSON, 1997).

In this context, it is important to develop methodologies capable of overcoming the limitations inherent to traditional methods. A successful example is the Statistical Energy Analysis (SEA), which allows the solution of dynamic problems through a division of the system under analysis in several subsystems and then, takes a statistical approach as a basis to evaluate mean values of response and energy. The SEA-based approach has greater applicability to medium and high frequencies because, when taking into account systems with many degrees of freedom, the energy contribution of individual eigenvalues (natural frequencies) and eigenvectors (vibrational modes) have less significance on the overall system response due to the high number of modes per frequency, (high modal density) (LYON and DEJONG, 1995; LYON, 1969; MACE, 2003, LEBOT, 2015, LANGLEY, 2007).

Well-consolidated studies have proven the strong relationship between the levels of mechanical vibration and the propagation of structural-borne sound (WHITE and WALKER, 1982, NILSSON AND LIU, 2015). In addition, one of the most efficient ways to reduce vibration levels in complex structures evolves the treatment of the source. Thereby, to reduce noise levels, it is necessary to control the vibration at the source.

Since MEF and SEA are recommended for applications in different frequency ranges. Thus, a complete solution for vibroacoustic parameters of a system requires the use of a methodology which combines the advantages of both methods. Recent studies have focused on the determination of the critical path of vibration transmission using hybrid methods (YANG and LI, 2010; EDWIN et al, 2012; CERECEDA et al, 2013). An important work explore the structural optimization of systems from the vibroacoustic point of view (CULLA et al, 2016). These studies show that hybrid methods are adequate to characterize vibroacoustic parameters over a wide frequency range.

Based on the studies mentioned, the present article aims to compare certain vibrational parameters, obtained from FEM and SEA approaches, such as modal density, modal overlap factor and vibrational energy, for different geometric arrangements of thin metal sheets. The plate arrangements represent engine beds which are commonly used to couple the hull of vessels to the main engine. The type of coupling considered here is mainly made by rigid welded joints, which actually increase the transmission of vibration. The geometries tested are based on the work of Lin (2005). Based on the results of vibrational energy transmitted through each of the arrangements tested, their effectiveness was evaluated in terms of an attenuation coefficient .

2. THEORETICAL BACKGROUND

Energy Statistical Analysis is based on the subdivision of complex structures into smaller fundamental structures, called subsystems. These subsystems have common characteristics such as modal density (n_i) and damping loss factor (η_i). In contrast to deterministic methods, SEA performs a statistical approach to the calculation of average amounts of energy for each of the subsystems considered (ALLEN, 2006).

The vibration energy E of a subsystem depends on the type of motion to be considered. Thus, it can be considered that, for a permanent motion under a given random excitation, its energy will be composed of a kinetic and a potential part. However, considering a simple damped oscillator, the potential energy will be equal to the kinetic energy (BURROUGHS et al., 1997). Therefore, the total energy of the system can be written as in the Equation 1.

$$E = M\left\langle v^2 \right\rangle \tag{1}$$

Where M represents the mass of the subsystem and $\langle v^2 \rangle$ is the time quadratic mean of the vibration velocity. It is possible to extend Eq. 01 to a discrete system considering the velocities of a finite number of points N, thus generating the Equation 2 [LYON and DEJONG, 1997].

$$E = M\left[\frac{1}{N}\sum_{i=1}^{N}\left\langle v^2 \right\rangle\right] \tag{2}$$

Other determinant parameter for SEA analysis is the modal density, n_i. In summary, this parameter represents the number of modes per unit of frequency and depends on geometric parameters and properties of each subsystem. For plate-like subsystems, modal density is defined according to the Equation 3 (LYON and DEJONG, 1997):

$$n_i(f) = \frac{S_i\sqrt{3}}{c_l h} \tag{3}$$

Where S_i is the area of the subsystem, c_l is the longitudinal wave propagation velocity and h is the plate thickness. The modal density can also be obtained using the subsystem response to a defuse excitation and it can be calculated by the Equation 4:

$$n_i(f) = 4M\left\langle \mathrm{Re}\{Y\} \right\rangle_x \tag{4}$$

Where Y is the punctual mobility and $\langle \mathrm{Re}\{Y\} \rangle_x$ represents the spatial average of real part of the mobility. The importance of the modal density in SEA analysis relies on the premise that the higher the number of modes sharing the energy, flatter is the system response and therefore it approaches a mean value. One way to estimate how much the modal density is appropriate to a statistical analysis is through the estimation of the Modal Overlap Factor (MOF). This parameter is evaluated based on the modal density of the subsystem (n_i), the damping loss factor (η_i) and the frequency of analysis ω, as in the Equation 5.

$$MOF_i = \omega \eta_i n_i \tag{5}$$

The MOF estimates how dependent of a single mode the response at a certain band of frequency is. Regarding, frequency bands with low MOF, it is likely that a single vibrational mode is dominant over the others. The bigger the MOF, the higher is the number of modes that contribute to the response of said band.

3. METHODOLOGY

Based on the methodology employed by Lin (2005) (arrangements C1, C2 and C3), simplified plate arrangements were considered to exemplify a generic engine bed. This consists of basically three main subsystems, namely P1, P2 and P3, where P1 represents the location where the vessel engine is installed, P2 represents the support structure between P1 and P3, this last one representing the vessel hull (Fig. 1).

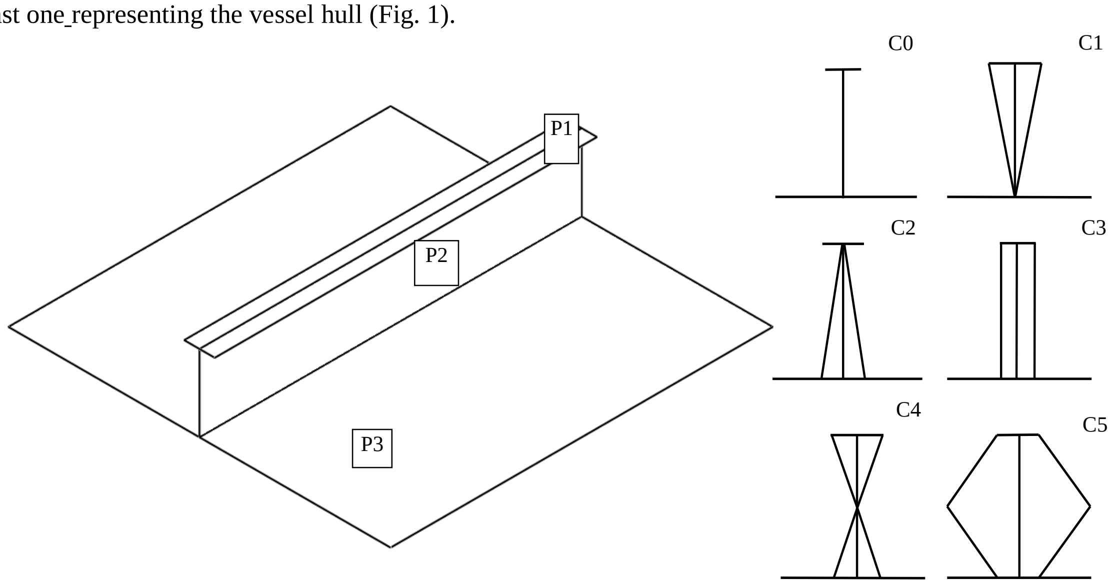

Figure 1. System under analysis and different arrangements proposed.

The arrangement is composed of 3.14 mm thick sheet steel plates. The material has a specific mass of 7700 kg / m^3, and elasticity modulus and Poisson's coefficient equal to 200 GPa and 0.3, respectively. The damping loss factor was estimated as a constant value of 0.5 %. To connect the subsystems, a line coupling was adopted to emulate the welding join.

The Finite Element simulations were performed using the ANSYS software. The discretization was performed using 12 SHELL188 elements per wavelength, resulting in meshes with the number of elements ranging from 12470 to 18692, respectively, for the six mentioned configurations. Harmonic simulations were carried out using as excitation a unitary force applied to 3 different locations of P1. More than 1 excitation point is needed to excite all possible modes of the structure. Then, for each subsystem, 8 uniformly spaced nodes were randomly chosen as measurement points. The frequency discretization adopted was 8 Hz, selected according to a previous convergence study. The range of interest ranged from 0 Hz to 1600 Hz. The SEA approach was performed using the commercial VAOne software.

4. RESULTS

The modal densities of each subsystem were obtained using the Equation 3 and are shown in Fig. 2. One can be observed that the modal density of P3, on average, is about 10 times bigger than of P1. This was expected since the modal density is directly proportional to the surface area of the component.

It is clear that the modal density obtained through the FEM response has a high level of divergence at low frequency region and as the frequency increases, it converges to the values obtained using the analytical equation. It can be attributed to the higher number of modes and this leads to a flatter response over a broader frequency band.

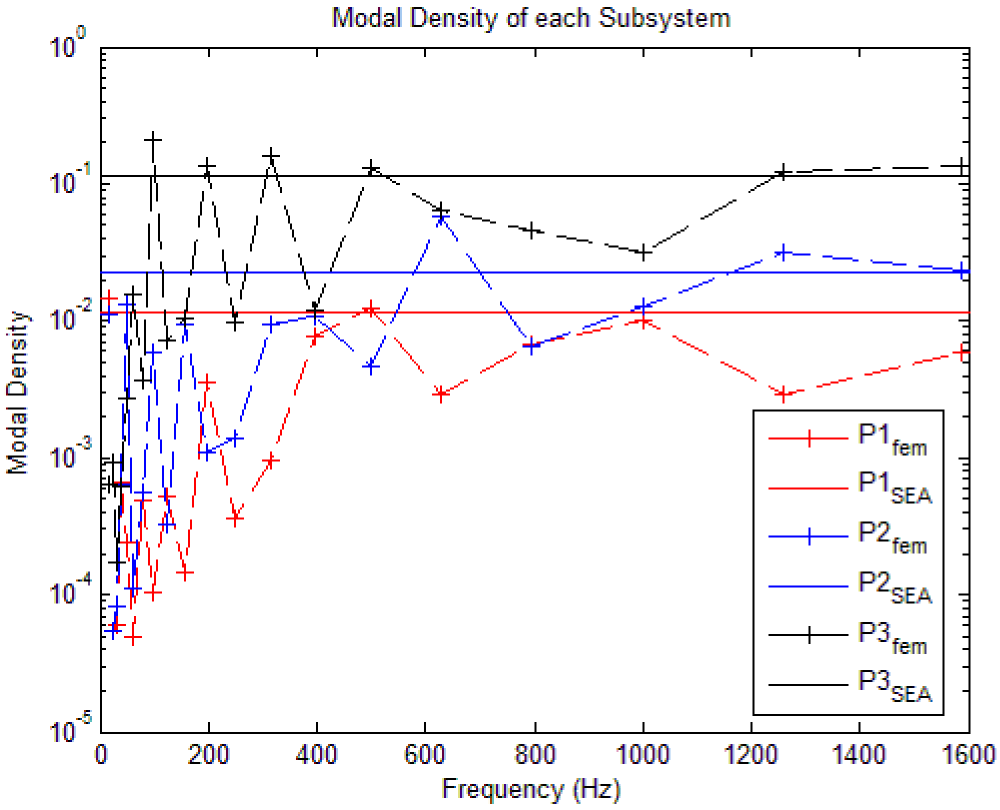

Figure 2. Modal density of each individual subsystem.

As a way to quantify how close the SEA approach will be to its fundamental formulation, the MOF parameter was evaluated (Fig. 3) using the Equation 5.

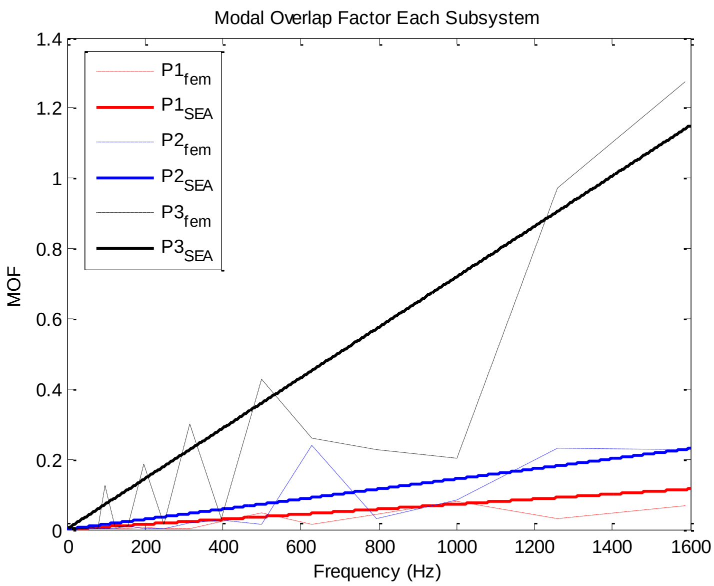

Figure 3. Modal Overlap Factor for each individual subsystem.

The trend between the MOF values, calculated by SEA and by FEM, behave similarly to the modal density evaluation, showing more accuracy as the frequency increases. It can be concluded that the subsystems P1 and P2, due to their low MOF values, have behavior dominated by single individual modes. On the other hand, the vibrational behavior of P3 is dictated by the contribution of a large number of modes.

With regard to the several geometries proposed, the energy was evaluated for the components P1 and P3 in order to determine a coefficient to express the attenuation of vibratory energy, calculated by the Equation 2, for each arrangement tested. This evaluation methodology proved to be adequate since it showed little deviation when employed along with the SEA approach. The Figure 3 presents the calculated energy for P1 and P3 considering the arrangement C0. It is clear that the energy level values, obtained from FEM and SEA, converge at high frequency since one of the key assumptions of SEA become more valid.

Additionally, one can be observed in Figure 4, that the energy calculated for P1 presents high deviations concerning the two approaches. This can be explained due to the low MOF values obtained for P1, as depicted previously in Figure 2. Since few modes dominate the response, it becomes more noticeable the existence of well-defined peaks and valleys. Still, the higher deviation of the energy in P1 can be attributed to the effect caused by a punctual excitation, since the greater the distance between the force application point and the analysis point, the higher is the contribution of modal superposition on the response due to that excitation.

In contrast, P3 has higher MOF values and it presents a linear response at high frequencies. At this region of the frequency spectrum, both methodologies converge to the same value.

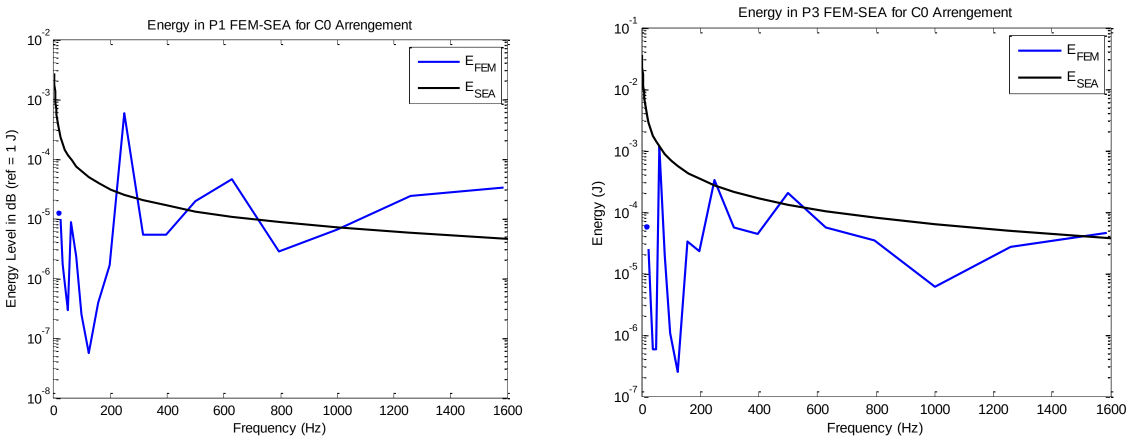

Figure 4. Energy calculated for the subsystems of the first arrangement: P1 (on left) and P3 (on right).

In general, most geometries presented behavior similar to that one observed for C0, except the C2 configuration, as depicted in Figure 5. This can be explained due to the fact this specific arrangement has only one path of energy transmission, being through P2, and the high stiffness over a certain frequency range provided by the attached components deviates from the mean response take into account on the analytical model**.**

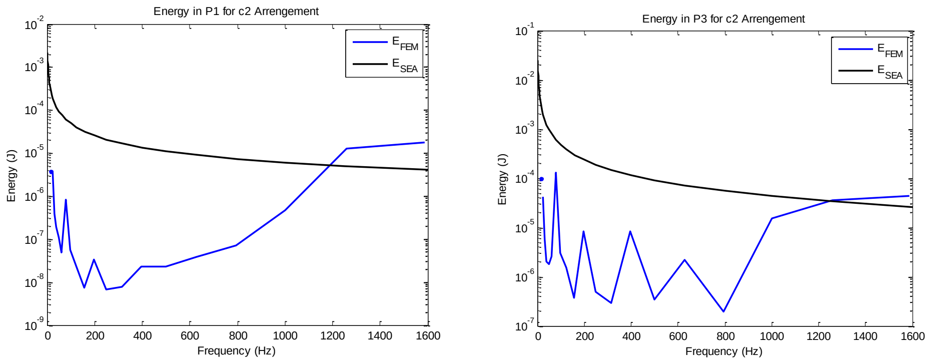

Figure 5. Energy for the arrangement C2.

In order to establish a relationship between E_{P1} and E_{P2}, it was proposed an attenuation coefficient, defined by the Equation 6.

$$R = 10\log_{10}\left(\frac{E_{P1}}{E_{P2}}\right) \tag{6}$$

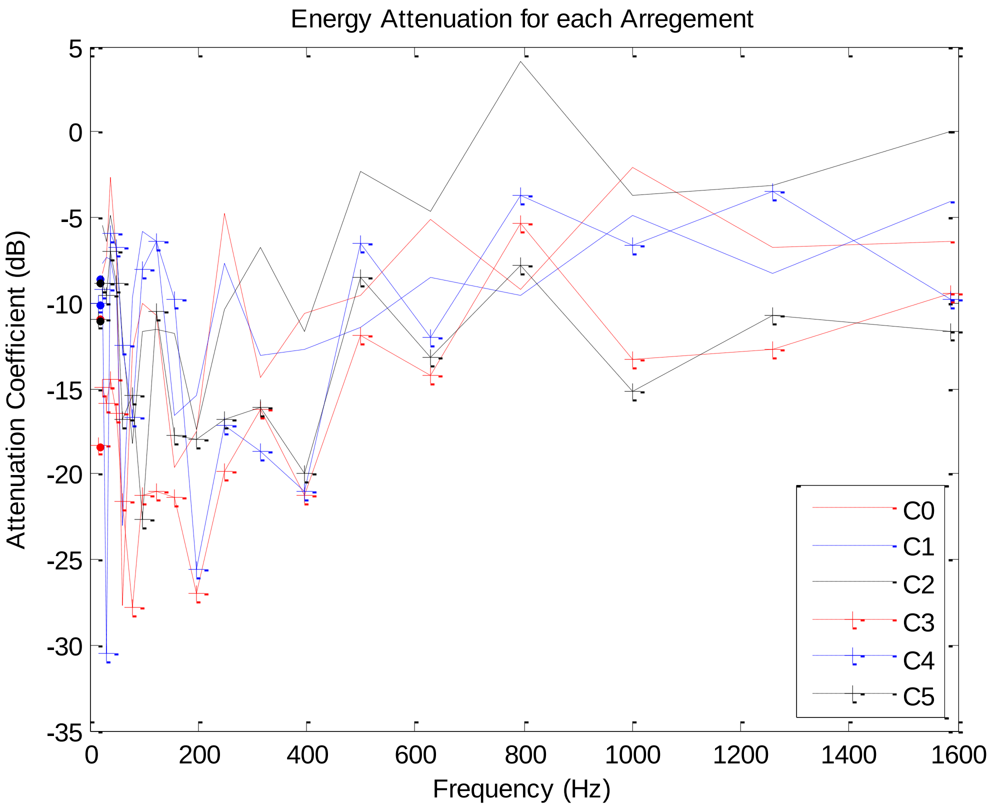

Figure 6. Attenuation coefficient for the different arrangements evaluated.

It is observed that the default configuration, C0, presents the worst attenuation due to the rigid coupling and lack of subsystems that can provide structural damping and/or insert energy loss at the coupling.

The C4 configuration, among all arrangements tested, is the one that presents the largest number of subsystems coupled with the main structure. This configuration not only it raises the damping loss factor associated with the main transmission path but also insert several coupling points and, consequently, insert losses by means of these couplings. This higher amount of couplings is also responsible for the high attenuation at low frequencies, since the coupling loss factor tends to be higher at the mentioned frequency range.

C3 also provides a large attenuation but only at high frequencies. This can be explained by the direct connection between P1 and P3 and, as frequency increases the additional subsystem presents a larger number of modes to share the same amount of energy, resulting in a larger dissipation at that frequency range.

5. CONCLUSIONS

This work compared analytical (SEA) and numerical (FEM) approaches to obtain major vibrational parameters (modal density, modal overlap and average energy) of a system composed by thin plates. Different arrangements of plates were considered to represent a structure commonly found in naval applications to couple the engine to its hull. The results proved some basic assumptions of SEA, in which it was observed that the modal density in plate-like structures is constant over frequency and it is directly proportional do the surface area of that subsystem. Additionally, it was found that the modal overlap factor is proportional to the both frequency and modal density. The agreement between SEA and FEM results was observed to increase with the modal overlap factor, since the overall response of the subsystem depends on the superposition of several modes rather than a single high-energy mode.

When comparing the different tested configurations, the C3 and C4 configuration showed a greater reduction in the transmitted energy between P1 and P3. C4 presented the largest attenuation at low and mid frequency ranges due to additional coupling joints and, thus contributing for further energy dissipation at connections. In this case, SEA analysis is highly recommended, since it showed hundreds of times faster than the Finite Elements Analysis. This can be aggravated when trying to model more complex geometries such as the full hull, which could require millions of degrees of freedom to be solved.

Finally, further research should be conducted concerning the scope of this topic, covering additional solutions such as: (i) to use materials, with viscoelastic properties, to be inserted between the subsystems, (ii) and to propose more detailed configurations to minimize the transmission of power from the source to the base of the structure.

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