Radio-Wave Propagation Predictions in a Three-Layered Medium for VHF/UHF based on Dyadic Green's Function

Diego K. N. da Silva, Cristiane R. Gomes, Jasmine Araújo, Herminio S. Gomes, Gervásio P. S. Cavalcante
Federal University of Pará, Belém, Pará, Brazil,
diegokasuo@ufpa.br, cris.ruiz.gomes@gmail.com, jasmine@ufpa.br, herminio.gomes@gmail.com, gervasio@ufpa.br

Abstract—**This article outlines a radio-wave propagation model for VHF/UHF based on the dyadic Green's functions. A three-layered medium was considered to characterize the path in a better way. Dyadic is a powerful tool for solving problems in the area of electromagnetism that allows properties of the medium to be incorporated. The study also takes account of the anisotropic medium. The proposed model has a good performance, presenting a small RMS error. A measurement campaign was conducted, together with comparisons with traditional models to validate the model.**

Index Terms—**Radio-wave propagation, Dyadic Green's function, VHF/UHF.**

I. INTRODUCTION

Planning for comunication systems is carried out in accordance with criteria that ensure the quality of the signal, e.g., the minimum field strength for reception with indoor antenna. Studies were needed to show the behavior of the transmitted signal at the time it reaches the receiver. Depending on the position of the receiver, the signal is degraded along the path for several reasons. The propagation loss can be analyzed through prediction models that decribe the electromagnetic signal between the transmitter and receiver.

The increase of communications systems has led to a large number of prediction methodologies and mathematical tools. These allow one to take into account the anisotropy and other characteristics of the medium for a better understanding of how to plan/implement these systems and improve their quality of service (QoS).

Dyadic Green's functions (DGF) has been used by many authors to analyze electromagnetic waveguides, resonant cavities and propagation in semi-infinite or layered media [1]-[5]. Other studies have examined its applicability for electromagnetic signal propagation at low frequencies, such as radio propagation services in the frequency range of 30 MHz to 200 MHz [6], [7].

DGF have served as a valuable and powerful tool for solving problems in electromagnetic scattering, radiation and propagation phenomena. It is a tool that allows properties of the medium to be incorporated, such as permittivity and conductivity for example. The calculation of the electric field of inhomogeneous anisotropic medium can be carried out in a less complex manner by using this tool.

A recent work [8] showed how DGF can be used to predict the electric field for VHF and UHF systems, by considering a two-layered medium. A comparison of the electric field was made by means of DGF with the model used in Rec. ITU-R P.529-3 (a modified Okumura-Hata model) [9], and this showed its applicability to frequencies in the range 30 - 200 MHz and for a higher range.

DGF solutions were obtained from its expansion in eigen functions for the prediction of the electric field. Electromagnetic fields were computed in their entirety, within appropriate boundary conditions. The main advantages of this methodology are: i) the precision of expansion of the eigen functions (which is guaranteed by the spectral theory); ii) greater flexibility with regard to the characteristics of the medium; iii) the ability to embed fonts with a stream of arbitrary distribution; iv) its application in isotropic and/or anisotropic media.

This paper proposes a radio-wave propagation model for VHF/UHF based on DGF that employs a three-layered medium, in which more characteristics of the media are included. This increases the likelihood of a case being treated/studied. A comparison between the proposed model, the traditional Okumura-Hata and ITU-R P.1546-4 [10] models and real data was made to analyze and validate the model.

This paper is divided into five sections: Section II sets out the three layer model using DGF; Section III examines the measurement campaigns that were carried out; Section IV analizes the results and Section V summarizes the conclusions of the proposed work.

II. MODEL USING DYADIC GREEN´S FUNCTION

A dyad is the mathematical entity formed by any two vectors

$$\overline{\overline{\mathrm{D}}} = \bar{A}\bar{B} \tag{1}$$

where $\overline{\mathrm{A}}$ is the previous element and $\overline{\mathrm{B}}$ the subsequent element.

The notation $\overline{\overline{\mathrm{G}}}_{\mathrm{i}}^{(\mathrm{sf})}(\bar{R}/\bar{R}')$ with $s, f = 1,2,3, \ldots$ and $i = 1,2,3$ and 4 will be used to express the dyadic Green's function which is the solution of the following partial differential equation

$$\nabla^2 \bar{\bar{\mathrm{G}}}_{\mathrm{i}}^{(\mathrm{sf})}(\bar{\mathrm{R}}/\bar{\mathrm{R}}') + k^2 \bar{\bar{\mathrm{G}}}_{\mathrm{i}}^{(\mathrm{sf})}(\bar{\mathrm{R}}/\bar{\mathrm{R}}') = \begin{cases} \bar{\bar{I}}\delta(\bar{R} - \bar{R}'), s = f \\ \quad 0 \quad\; , s \neq f \end{cases} \tag{2}$$

where $\bar{\bar{I}}$ is the dyadic unit and δ refers to the Dirac delta.

This work employs a structure of N-layers, (Fig.1), with the current source located at $(0, 0, z')$ of a cylindrical coordinate system. The superscripts (s, f) used in the equations are combined with the medium containing the observation point and the position of the source, respectively. The subscript **i** is related to the type of Green's function.

In this study, the electrical properties of the medium are defined by the complex permittivity $\varepsilon_r \varepsilon_0 = \varepsilon' + j\frac{\sigma}{\omega}$ and the magnetic permeability μ will be considered to be the same as the free-space in all cases. For purposes of convenience these properties will be included in the propagation constant given by

$$k = \omega\sqrt{\mu_0 \varepsilon\,(1 + j\frac{\sigma}{\omega\varepsilon})} \tag{3}$$

The current source and the associated electromagnetic field are considered to have a variation with time $(e^{-j\omega t})$, which will be implied in this work.

For the calculation of the electric field, a current source $(\bar{\mathbf{J}}_{\mathbf{H}})$ shown in Fig. 2 is considered, when making the calculation of the current field. In this situation, a medium with three layers is considered, since the media include the following: medium 1 has a propagation constant k_1, magnetic permeability μ_0 and permittivity $\varepsilon_1 = \varepsilon_0$ and conductivity σ_1 (the same of free space); medium 2 has a propagation constant k_2, magnetic permeability μ_0 permittivity ε_2 and conductivity σ_1; medium 3 is lossy dielectric, homogeneous with constant propagation k_3, magnetic permeability μ_0, permittivity ε_3 and conductivity σ_3. The electric field $(\bar{\boldsymbol{E}}_{\boldsymbol{H}})$ is calculated for a far field at any point $(\bar{\mathbf{R}})$ of space. The observation point $(\bar{\mathbf{R}})$ – which is identified by the usual cylindrical coordinates – is at $(\rho, \emptyset, z)$. In the type of propagation problems considered here, (which for analytical purposes are restricted to the far field), it can be assumed that the magnitudes are much smaller than that of ρ; the radial distance is the distance between the current source and the observation point.

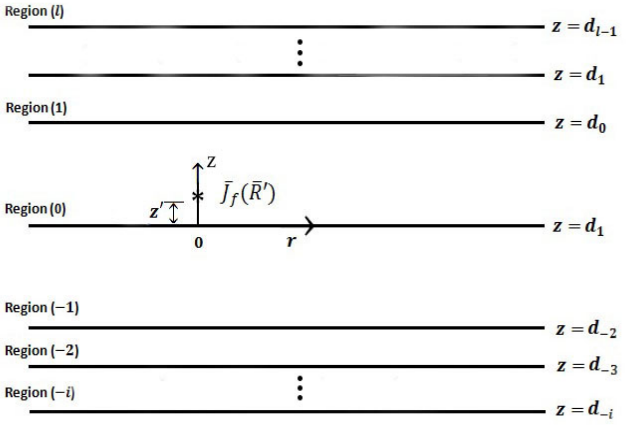

Fig. 1. Geometric configuration of the structure of N-layers.

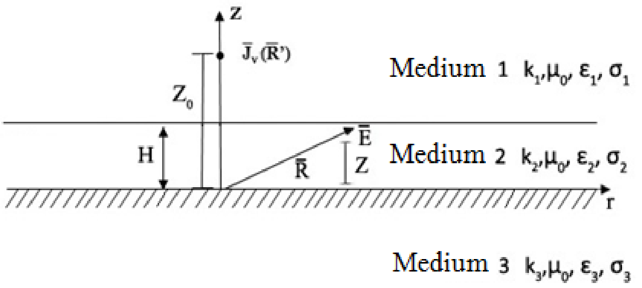

Fig. 2. Graphical representation of the situation.

The propagation constants in the media are

$$k_1 = \omega\sqrt{\mu_0\varepsilon_0}\;;\; k_2 = \omega\sqrt{\mu_0\varepsilon_2(1 + j\frac{\sigma_2}{\omega\varepsilon_2})}$$
$$k_3 = \omega\sqrt{\mu_0\varepsilon_3(1 + j\frac{\sigma_3}{\omega\varepsilon_3})} \tag{4}$$

Medium 1 is considered to be the air, with characteristics of permittivity and conductivity, given by ε_1, δ_1; respectively, the second medium, e.g. the buildings of a city or the trunk of a forest tree, has characteristics given by ε_2, δ_2 and a third medium, in this case the soil, has characteristics given by ε_3, δ_3,

In this situation, where the current source is in medium 1, the observation point $(\bar{\mathbf{R}})$, in which the electric field is calculated, it follows that Green's function will be a function [4] of a variable λ, given by

$$\bar{\bar{G}}_3^{(21)}(\bar{R}/\bar{R}') = \frac{j}{4\pi}\int_0^\infty \frac{d\lambda}{\lambda h_1}\sum_{n=0}^{\infty}(2 - \delta_0)\cdot\{[a\bar{M}_{\substack{e\\o}n\lambda}(h_2) + b\bar{M}_{\substack{e\\o}n\lambda}(-h_2)]\bar{M}'_{\substack{e\\o}n\lambda}(h_1) + [c\bar{N}_{\substack{e\\o}n\lambda}(h_2) + d\bar{N}_{\substack{e\\o}n\lambda}(-h_2)]\bar{N}'_{\substack{e\\o}n\lambda}(h_1)\} \tag{5}$$

where

$$\mathrm{h}_1 = \sqrt{\mathrm{k}_1^2 - \lambda^2} \tag{6}$$

$$\mathrm{h}_2 = \sqrt{\mathrm{k}_2^2 - \lambda^2} \tag{7}$$

$$\bar{M}_{\substack{e\\o}n\lambda}(h) = \left[\pm\frac{nJ_n(\lambda r)}{r}\,{}^{sin}_{cos}(n\Phi\hat{r}) - \frac{\partial j_n(\lambda r)}{\partial r}\,{}^{cos}_{sin}(n\Phi\hat{\Phi})\right]e^{jhz} \tag{8}$$

$$\bar{N}_{\substack{e\\o}n\lambda}(\mathrm{h}) = \frac{1}{k_\lambda}\left[jh\frac{\partial j_n(\lambda r)}{\partial r}\,{}^{cos}_{sin}(n\Phi\hat{r}) \mp \frac{jhn}{r}J_n(\lambda r)\,{}^{sin}_{cos}(n\Phi\hat{\Phi}) + \lambda^2 \mathrm{J_n}(\lambda \mathrm{r})\,{}^{\mathrm{cos}}_{\mathrm{sin}}(\mathrm{n}\Phi\hat{\mathrm{z}})\right]e^{jhz} \tag{9}$$

a, ***b***, ***c*** and ***d*** are constants determined by the boundary conditions between the media (omitted here for convenience). Observing that J_n is the Bessel function of order n.

The electric field is calculated according to

$$\bar{E}_H(\bar{R}) = j\omega\mu_0 \iiint \bar{G}^{(21)}\left(\bar{R}/\bar{R}'\right)\cdot \bar{J}_H(\bar{R}')\,dV' \tag{10}$$

where the horizontal electric dipole is given by (13), using a current moment ρ.

$$\bar{J}_H(R') = \rho\delta(x-0)\delta(y-0)\delta(z-z_0) \tag{11}$$

Making use of (5), (8), (9) and (11) in (10), the model using dyadic Green´s function for calculating the electric field is given by

$$\begin{aligned}\bar{E}_H(\bar{R}) = -\frac{\omega\mu_0\bar{p}}{4\pi}\int_0^\infty \frac{e^{jh_1z_0}}{h_1}\Big\{&\big[a\bar{M}_{o_{1\lambda}}(h_2) \\ &+ b\bar{M}_{o_{1\lambda}}(-h_2)\big] \\ &+ \big[c\bar{N}_{e_{1\lambda}}(h_2) \\ &+ e\bar{N}_{e_{1\lambda}}(-h_2)\big]\frac{jh_1}{k_1}\Big\}\,d\lambda\end{aligned} \tag{12}$$

The complete evaluation of the field quantities given by (12) begins with the resolution of the integral form of dyadic Green’s function. A change of variable is applied to obtain the integration in terms of k_ρ and Bessel function. Since numerical computation of these expressions is slow and formidable, due to the presence of singularities and the fluctuating behavior of the Bessel function at far distance ρ, approximate asymptotic solutions are required. In a widely used standard procedure the first and second kind Hankel’s functions were used to calculate the electric field in the far field.

III. MEAUREMENT CAMPAIGNS

Measurement campaigns were carried out in Belém-PA city (1º27'18.62"S, 48°30'08.49"W), Brazil during 2013. Power and electric field data from two DTV stations were collected. The nominal frequency of the transmitter Tx1 is 521.14 MHz and for Tx2 is 515.14 MHz, (Table I).

Measurements were made at 75 points of Belém and covered the urban and suburban perimeter, with distances ranging from 1km to 20km, (Fig.3).

The measurement setup involved installing an antenna Anritsu MPP651A dipole for the frequency range 470 MHz to 1700 MHz on the roof of a vehicle, with the aid of an aluminum tripod. A 3 m long cable connected the antenna to the portable spectrum analyzer Site Master S332E (also an Anritsu) inside the vehicle.

TABLE I. INFORMATION ABOUT THE TRANSMITTING ANTENNAS

Transmitter	**Tx1**	**Tx2**
Location	01º27’43”S/4 8º29’28”W	01º27’12”S/ 48º29’22”W
Height (m)	114.58	125.30
Range (MHz)	518-524	512-518
Power Operation (kW)	6.00	10.00
Effective Radiated Power (ERP) (kW)	52.15	61.79

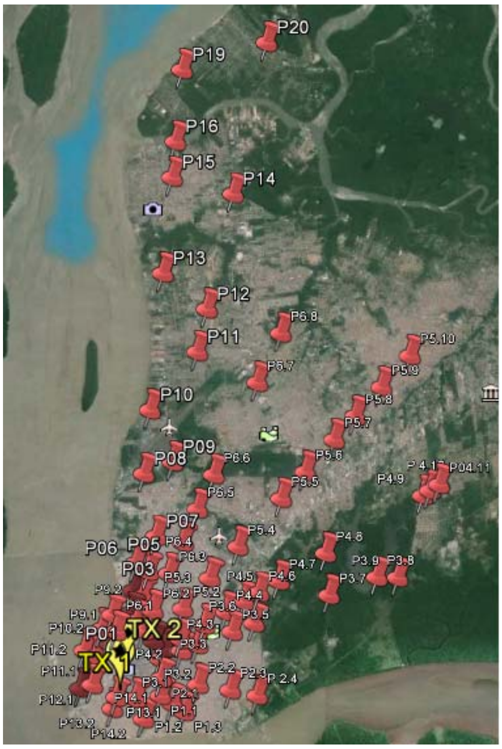

Fig. 3. Display of measured points in Belém.

IV. RESULTS

The proposed model was encoded in Matlab®. The inputs of the model are the characteristics of the transmitting and receiving antenna, the electric constant and the wave propagation constants of the medium. Its output is a function (exponential tendency) that describes the electric field.

The DGF model was compared with the measured data and two well-known models: Okumura-Hata and ITU-R P.1546-4.

For simulation, the parameters used for the Medium 2 and 3 were tabulated values for a medium with similar characteristics to the cities [11]. For example, for medium 2 were used $\sigma = 40\, mS/m$ and $\varepsilon_r = 2.7$, respectively, for conductivity and relative permittivity for a road. A change in the parameter values leads to a modification of the graphs generated by simulation, because depending of the medium, the refractive index will be changed and there will be higher or lower absorption of the electromagnetic wave by the soil considered.

Fig.4 illustrates the power level for the three different models and for the measured data for Tx1. Analogously Fig.5 illustrates the condition for Tx2.

In the two DVT stations, the DGF model has a better performance following the average of measured data.

For Tx1, the three models were fairly good and had RMS errors of less than 6 dB. However the traditional models were not so good for the Tx2 case, and underestimated the power level as was the case in other works [12], [13].

A comparison of RMS errors can be seen in Table II. In both cases DGF showed a smaller RMS (less than 5 dB).

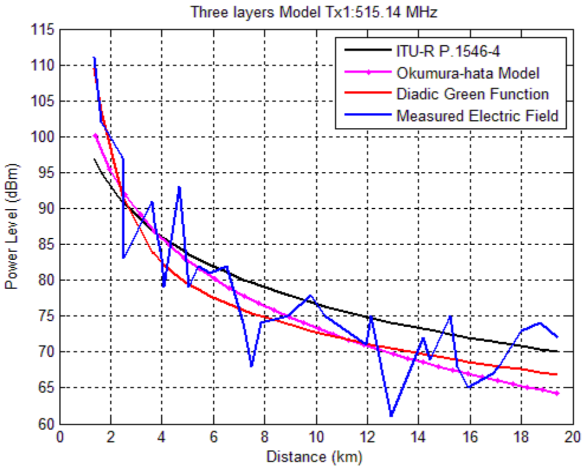

Fig. 4. Measured power level data, traditional models and the proposed model for Tx1.

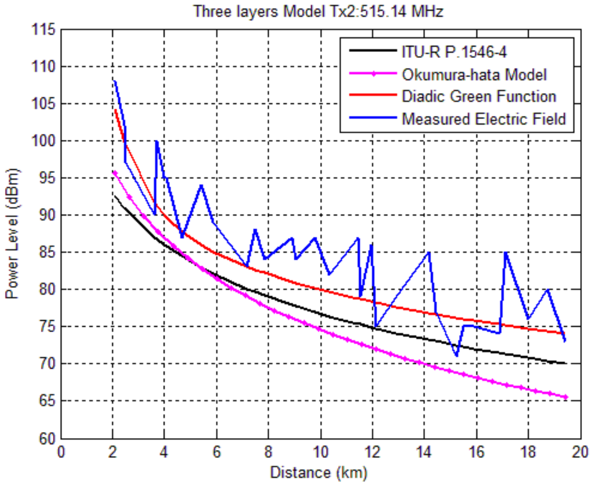

Fig. 5. Measured power level data, traditional models and the proposed model for Tx2.

TABLE II. COMPARISON BETWEEN THE MODELS

Model	**RMS errors**	
	Tx1	***Tx2***
Okunura-Hata	5.7559	8.0504
ITU-R P.1546-4	5.5212	10.3775
DGF	4.8229	4.7899

V. CONCLUSION

A radio-wave propagation model for VHF/UHF based on DGF was proposed in this paper that considers a three-layered medium. This tool allows some properties of the medium in question to be incorporated - characteristics such as permittivity and conductivity for example. The dyadic system also enables the anisotropy of the medium to be considered.

DGF was used to compute the electric field for systems operating at higher frequencies.

The results showed that the model is consistent. The evaluation of the model found in *Rec. ITU-R P.529-3* and *Rec. ITU-R P. 1546* showed the proposed formula was applicable to the electric field.

In future works, studies will be carried out with multilayered anisotropic media. Our aim is to employ the proposed model in different scenarios and frequency bands. A further study that is planned seeks to add the use of Bayesian Networks with DGF to obtain more accurate values of electrical parameters of the ground.

ACKNOWLEDGMENT

The authors would like to thank the National Institute of Science and Technology-Wireless Communication (INCT-CSF) for its support.

REFERENCES

[1] P. Ding, C. W. Qiu, S. Zouhdi, S. P. Yeo, “Rigorous derivation and fast solution of spatial-domain Green’s functions for uniaxial anisotropic multilayers using modified fast Hankel transform method,” IEEE Trans Microw Theory Techn., vol. 60, no. 2, pp. 205-217, Feb. 2012.

[2] A. Fallahi, B. Oswald, “On the computation of electromagnetic dyadic Green’s function in spherically multilayered media,” in IEEE IEEE Trans Microw Theory Techn.,vol. 59, no. 6, pp. 1433-1440, Jun. 2011.

[3] P. M. Morse, and H. Feshbach, “Methods of Theoretical Physics”, Parts 1 and 2, pp. 52-54, 1759-1901, Mc Graw-Hill, N. Y., 1953.

[4] C. T. Tai, “Dyadic Green’s Functions in Electromagnetic Theory”, Intext, Scranton, 1971.

[5] G. P. S. Cavalcanti, David Anthony Rogers, Attílio José Giarola, “Analysis of electromagnetic wave propagation in multilayered media using dyadic Green’s functions”, in Radio Science, Volume 17, Number 3, pages 503-508, May-June 1982.

[6] D.H. Liaoand K. Sarabandi,“ Near-Earth Wave Propagation Characteristics of Electric Dipole in Presence of Vegetation or Snow Layer”, IEEE Transactions On Antennas and Propagation, vol. 53, no. 11, november 2005.

[7] Le-Wei Li, Chee-Keong Lee, Tat-Soon Yeo, Mook-Seng Leong,”Wave Mode and Path Characteristics in a Four-Layered Anisotropic Forest Environment”, IEEE Transactions On Antennas and Propagation, vol. 52, no. 9, september 2004.

[8] D. K. N. Silva, W. J. da S. Barros, J. P. L. de Araújo, G. P. S.Cavalcante, “Electric Field Prediction for VHF and UHF Systems Using Dyadic Green's Functions”, in 16th SBMO - Brazilian Symposium on Microwave and Optoelectronics/11th CBMag - Brazilian Congress of Electromagnetism, september 2014.

[9] Prediction Methods for the Terrestrial Land Mobile Service in the VHF and UHF Bands. RECOMMENDATION ITU-R P.529-3.

[10] Method for point-to-area predictions for terrestrial services in the frequency range 30 MHz to 3 000 MHz. RECOMMENDATION ITU-R P.1546-4.

[11] G. P. S. Cavalcante, M. A. R. Sanches, and R. A. N. Oliveira, “Mobile Radio Propagation along Mixed Paths in Forest Environment”, Proceedings of IMC´99, SBMO/IEEE, 1999, pp. 320-324.

[12] M. A. S. Mayrink, F. J. S. Moreira, C. G. Rego, L. A. R. Silva Mello, Evaluation of Recommendation ITU-R P1546 for UHF Field-Stregth over Fresh-Water Mixed Paths, in 12th SBMO - Brazilian Symposium on Microwave and Optoelectronics/7th CBMag - Brazilian Congress of Electromagnetism, August 2006.

[13] C. R. Gomes, R. N. A. Nascimento, H. S. Gomes, G. P. S. Cavalcante, “Methodology Based on Recommendation ITU-R P.1546-4 for Prediction of Field-Strength in Mixed Path: Land-Freshwater” in 15th SBMO - Brazilian Symposium on Microwave and Optoelectronics/10th CBMag - Brazilian Congress of Electromagnetism, August 2012.